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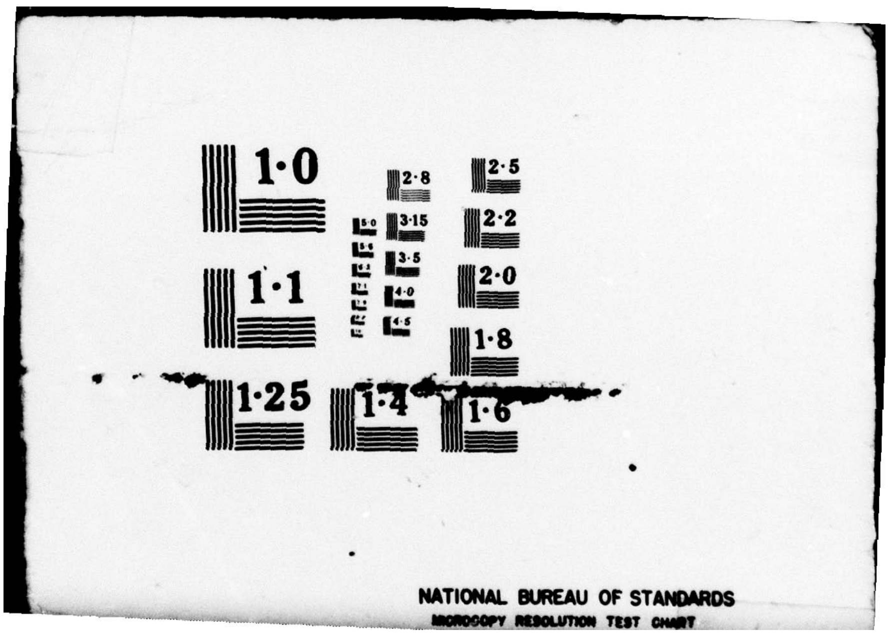
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ADAPTIVE FILTERING IN THE FREQUENCY DOMAIN USING THE COMPLEX LMS ALGORITHM

Mauro Dentino, John McCool, and Bernard Widrow

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Adaptive filtering in the frequency domain can be accomplished by Fourier transformation of the input signal and independent weighting of the contents of each frequency bin. The frequency-domain filter performs similarly to a conventional adaptive transversal filter but promises a significant reduction in computation when the number of weights equals or exceeds 16.

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INTRODUCTION

Adaptive filters are being used in a wide variety of applications, such as statistical prediction [1,2], interference cancellation [3], adaptive antennas [4], and channel equalization [5] in communication systems. The most widely used adaptive filters at the present time are nonrecursive adaptive transversal filters [1,4,5,6], although rudimentary forms of adaptive feedback filters are beginning to appear [7,8]. This paper presents a new approach to adaptive filtering that promises great improvements in computational efficiency from doing the entire process in the frequency domain.

CONVENTIONAL ADAPTIVE FILTERING

A "conventional" adaptive transversal filter is represented symbolically in Fig. 1. Details of this filter are shown in Fig. 2. If the sample time is j, the discrete input signal is represented by x_j. The filter output is y_j. The "desired-response input" is d_j. The latter input is a training signal necessary for effecting the adaptive process. Some ingenuity is generally required to obtain this signal in practice.

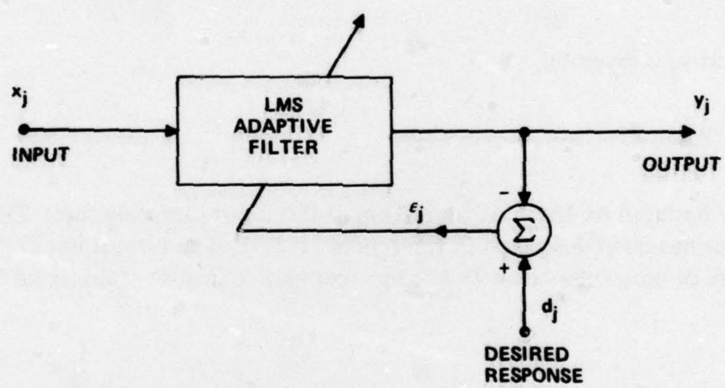

Figure 1. LMS adaptive transversal filter.

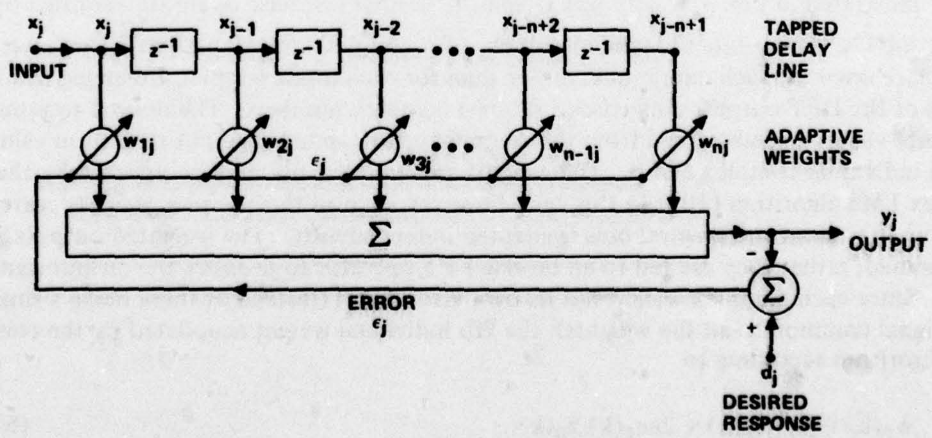

Figure 2. Details of LMS adaptive transversal filter.

In each of the applications mentioned above, the most commonly used adaptive algorithm at present is the LMS algorithm of Widrow and Hoff [6,9],

$$\underline{W}_{j+1} = \underline{W}_j + 2\mu\epsilon_j \underline{X}_j \,. \tag{1}$$

$\underline{W}_j$ is a column vector of filter weights at the time of the jth iteration cycle,

$$\underline{W}_j^T = [w_{1j}, w_{2j}, \ldots, w_{nj}] \,. \tag{2}$$

The input signal vector $\underline{X}_j$ is the set of values of the signals at the delay line taps of the adaptive transversal filter,

$$\underline{X}_j^T = [x_j, x_{j-1}, \ldots, x_{j-n+1}] \,. \tag{3}$$

The error ϵ_j is the difference between the desired response and the actual response,

$$\epsilon_j = d_j - y_j, \tag{4}$$

where the output y_j is given by

$$y_j = \underline{X}_j^T \, \underline{W}_j. \tag{5}$$

The weights are updated by the LMS algorithm at the input sampling rate. The functioning of the LMS filter has been described in references [1,3,6]. The term μ in (1) is a constant that governs rate of convergence and whose proper choice insures stability of the adaptive process.

ADAPTIVE FILTERING IN THE FREQUENCY DOMAIN

An alternative to the "time domain" LMS filter is the "frequency domain LMS filter," illustrated in Fig. 3. The input x_j and the desired response d_j are transformed by n-point DFT's. Thus, data are taken in blocks of n samples, and adaptation of each weight takes place once for each data block, rather than for each input sample. Referring to Fig. 3, each of the DFT outputs comprises a set of n complex numbers. The desired response transform values are subtracted from the frequency-corresponding input transform values to form n individual complex errors. The weights are complex and may be updated by the complex LMS algorithm [10]. In this case, however, each of the complex weights corresponding to each of the spectral bins is adapted independently. The weighted outputs are not summed; rather they are fed to an inverse FFT operator to produce the output signal.

Since each complex weight has its own error signal (instead of there being a single error signal common to all the weights), the ℓth individual weight is updated by the complex LMS algorithm according to

$$w_\ell(k+1) = w_\ell(k) + 2\mu\epsilon_\ell(k)\,\overline{x}_\ell(k) \,, \tag{6}$$

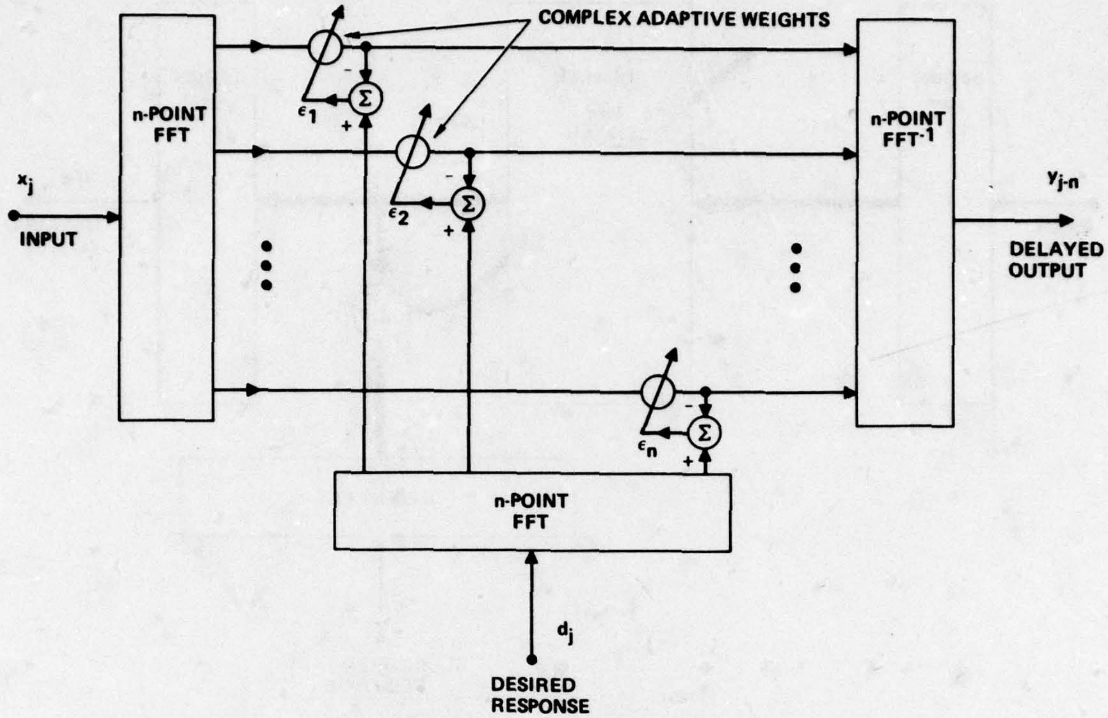

Figure 3. LMS adaptive filtering in the frequency domain.

where $w_\ell(k+1)$ is the ℓth complex weight after adaptation with the kth n-point input data block. Note that $\bar{x}_\ell(k)$ is the conjugate of $x_\ell(k)$. To produce output data similar to those produced by the transversal filter of Fig. 2, the number of weight adaptations is reduced by a factor of n since each weight is adapted only once for each n-point input data block. The value of the adaptive constant μ should accordingly be increased by a factor of n compared to the value of μ chosen for the scheme of Fig. 2, so that the rate of convergence and performance in general of the conventional and the frequency domain adaptive schemes would be comparable. A larger weight increment with each adaptation, corresponding to less frequent adaptations, would permit a lowering of the weight resolution requirements for the frequency domain scheme by a factor of n, so that the number of bits used to store each weight could be reduced by $\log_2 n$, simplifying the weight update arithmetic. Fig. 4 is a symbolic representation of the frequency domain filter of Fig. 3.

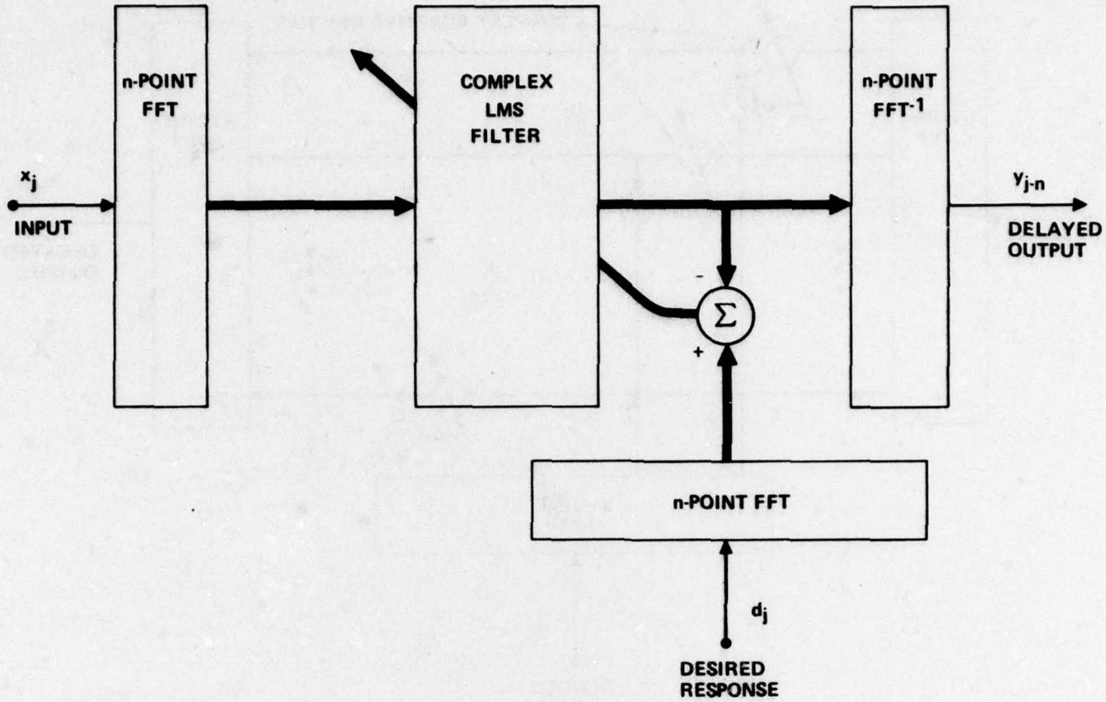

Figure 4. Symbolic representation of frequency domain adaptive filtering.

COMPARISON OF COMPUTATIONAL REQUIREMENTS

The adaptive transversal filter of Fig. 1 works on a continuous flow basis. The frequency domain filter of Fig. 4 in turn produces n points of output data all at once, based on the block of n input data samples. The effect of steady flow can be achieved by outputting in sequence the results of processing data from input block to input block. In order for the adaptive transversal filter to produce n output data samples and to be continually updated, with all of its n weights being adapted to each sample time, the number of weight adaptations is n^2. On the other hand, to achieve essentially the same result with the frequency domain filter, three n-point FFT's must be performed in addition to adapting each of the n complex weights just once.

One way of comparing the computational complexity of the time domain and frequency domain approaches to adaptive filtering is in terms of number of multiply operations. To produce n output data points and n^2 adapts, the adaptive transversal filter requires $2n^2$ real multiplies. To do the corresponding job in the frequency domain, three n-point FFT's require $3n \log_2 n$ complex multiplies, while the complex weighting and weight updating requires 2n complex multiplies. The ratio of complex multiplies required by the frequency domain filter to real multiplies required by the conventional filter is thus

$$\frac{\text{complex multiplies}}{\text{real multiplies}} = \frac{(3n/2)\log_2 n + 2n}{2n^2} = \frac{3\log_2 n + 4}{4n} \tag{7}$$

With n = 4, this ratio is 0.833; with n = 16 it is 0.250; with n = 128 it is 0.0469; and with n = 1024 it is 0.0083. With n = 4 there is thus no reduction in computational requirements, while with n = 16 the potential reduction factor is approximately 4, and with n = 1024 it is greater than two orders of magnitude.

It is apparent that in many practical cases the savings resulting from use of the frequency domain technique are substantial, so much so that, even when one takes into account the additional computing of 3 FFT's per data block, it pays to use this technique and to recover the filtered signal from its DFT. Using conventional digital techniques, the output could be "stitched together," creating a continuous data flow from blocks of n output data points. The output would be delayed in time, however, by at least the block length. On this basis, any large adaptive digital filter could be efficiently realized by a combination of the complex LMS and FFT algorithms. If one required only the DFT of the output or its power spectrum, the third FFT computation could be eliminated.

One application warrants special mention, that of detection of narrow-band signals buried in noise. The "adaptive line enhancer" (ALE) has been described and analyzed in references [3,11]. Griffiths [12] has shown that the ALE is capable of computing "maximum entropy" power spectra [13]. Fig. 5 shows a block diagram of an ALE based on adaptive filtering in the frequency domain. In this configuration, if the delay Δ is chosen to be an integral multiple of the FFT window width (n sample periods), the same FFT of the adaptive-filter desired response can also be used for the filter input, using inexpensive memory instead of computation of an additional FFT.

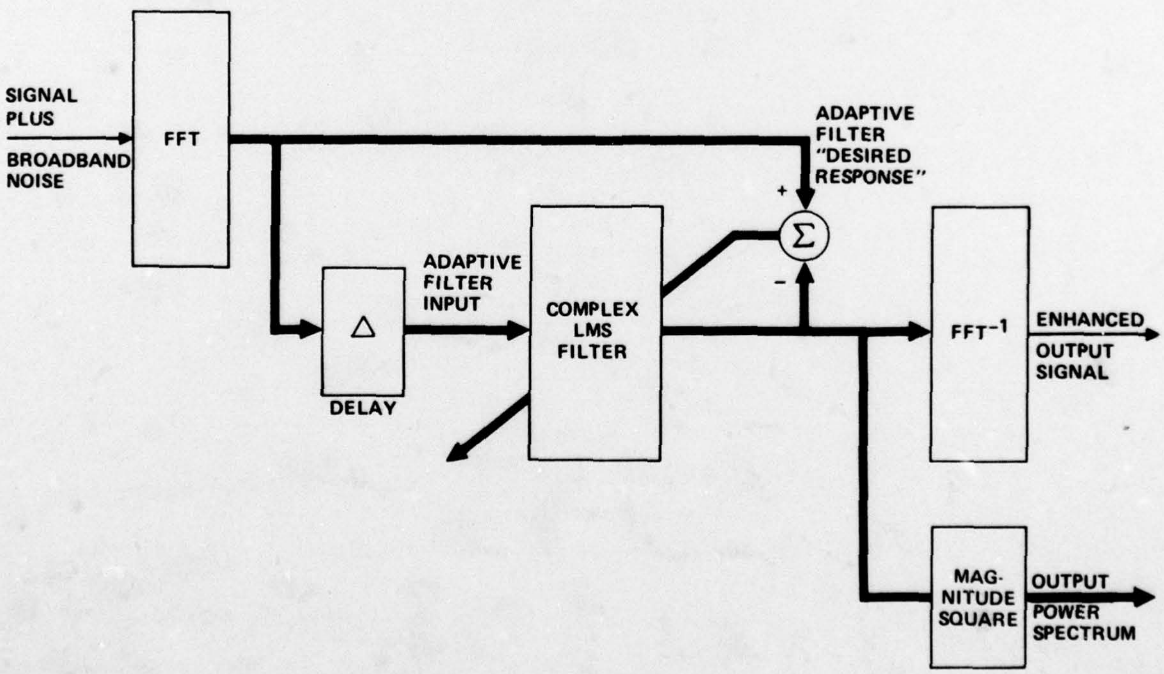

Figure 5. Frequency domain adaptive line enhancer.

CONCLUSION

The advantages of the frequency domain approach over conventtional time domain methods is very apparent for large n. The number of multiplies and the memory word length for the storage of the adaptive weights is greatly reduced. The number of bits carried through the weight update arithmetic is also greatly reduced. On the other hand, the number of memory registers required to store weights and data is significantly increased, but memory is cheap.

The results of adaptive filtering in the frequency domain are in most cases quite similar but not identical to those of adaptive filtering in the time domain. The nature of the differences and similarities are under study and will be reported in the future.

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